Coupled-channel effects in kaon pair production*

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Two coupled channel effects connected with kaon pair production in proton-proton collisions are discussed. (1) Although there is ample evidence that the antikaon is strongly attracted to the recoil protons in $pp \to K^+p\{K^-p\}$, residual effects of the K^+K^- interaction are seen, including a possible cusp at the $K^0\bar{K}^0$ threshold. This is investigated within a simple K-matrix approach. (2) The production rates and invariant mass distributions for $pp \to K^+p\{K^-p\}$ and $pp \to K^+p\{\pi^0\Sigma^0\}$ are related using a separable potential description of the coupled $K^-p/\pi^0\Sigma^0$ channels. It can be plausibly argued that this pair of reactions is driven through the production of the $\Lambda(1405)$.

The bulk of the observed distributions in $pp \to ppK^+K^-$ above and below the ϕ threshold can be understood in terms of pp and K^-p final state interactions [1, 2], as can be seen from Fig. 1. It is shown there that the K^-p *fsi* distorts particularly the ratio of the differential cross sections

$$R_{Kp} = \frac{\mathrm{d}\sigma/\mathrm{d}M_{K^-p}}{\mathrm{d}\sigma/\mathrm{d}M_{K^+p}}, \tag{1}$$

which has a very strong preference for low Kp invariant masses, M_{Kp}.

Since a full treatment of the dynamics of the four-body ppK^+K^- channel is currently impractical, the final state interactions were introduced in an *ad hoc* way, as the product of the enhancements in the pp and the two K^-p combinations, all evaluated at the appropriate relative momenta q [2]:

$$F = F_{pp}(q_{pp}) \times F_{Kp}(q_{Kp_1}) \times F_{Kp}(q_{Kp_2}). \tag{2}$$

This was used to generate the simulations shown in Fig. 1. The K^-p *fsi* was taken in the scattering length approximation, $F_{Kp}(q) \approx 1/(1 - iqa)$, where $|a| \approx 1.5$ fm. With this value of a, the approach reproduces the K^-p/K^+p ratio also at other energies [2], as well as the COSY-11 results [1]. Moreover,

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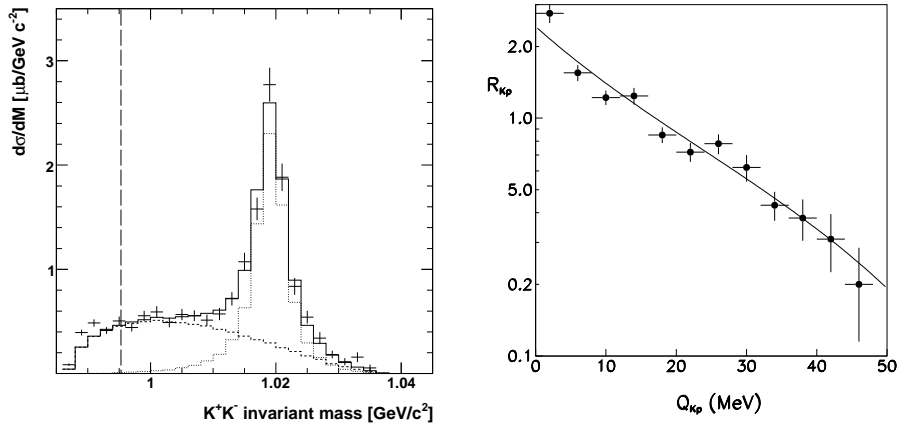* Presented at the Symposium on Meson Physics, Cracow 2008

Fig. 1. Left: Differential cross section for $pp \to ppK^+K^-$ at 2.65 GeV (crosses) as a function of the K^+K^- invariant mass compared to simulations of the ϕ (dotted) and non-ϕ (dashed) contributions and their sum (solid histogram). The $K^0\bar{K}^0$ threshold is indicated by the dashed vertical line. Right: The ratio of differential cross sections with respect to $Q_{Kp} = m_{Kp} - m_K - m_p$; see Eq. (1). The curve results from the amplitude analysis of Ref. [3], which includes a $\bar{K}^0d$ *fsi*.

the simulation suggests that the K^-pp system should be enhanced at low masses and this feature is also seen in the ANKE data [2]. Further evidence that the antikaon is attracted to nucleons is to be found in the $pp \to dK^+\bar{K}^0$ reaction, where low $\bar{K}^0d$ masses are favoured compared to K^+d [3].

However, this approach underestimates the data at low K^+K^- masses in Fig. 1 and so the *ansatz* of Eq. (2) has to be generalised to include an *fsi* in this channel. The effects are smaller here and, to illustrate them, the experimental data at all three ANKE energies have been divided by the simulations generated by Eq. (2), and their average is plotted in Fig. 2.

The enhancement seems to be most prominent between the K^+K^- and $K^0\bar{K}^0$ thresholds at 987.4 and 995.3 MeV/c^2, respectively. It is therefore natural to speculate that it is also influenced by virtual $K^0\bar{K}^0$ production and its subsequent conversion into K^+K^- through a charge-exchange *fsi*. If the *s*-wave $K^+K^- \rightleftharpoons K^0\bar{K}^0$ coupling is strong, this would generate an observable cusp at the $K^0\bar{K}^0$ threshold. These possibilities were examined in Ref. [4], where it was shown that the enhancement factor has a momentum
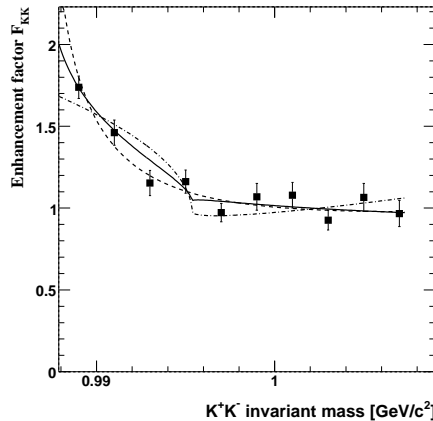
Fig. 2. Ratio of the K^+K^- invariant mass spectra from the $pp \to ppK^+K^-$ reaction to the simulation presented in Ref. [2]. The experimental points correspond to the weighted average of data taken at 2.65, 2.70, and 2.83 GeV. The solid curve is the result of a best fit of Eq. (3) to these data. The dot-dashed curve is the best fit when the elastic rescattering is *arbitrarily* neglected and the dashed when the charge-exchange term is omitted.

dependence of the form

$$\mathcal{F} = \left| \frac{B_1/(B_1 + B_0)}{\left(1 - i\frac{1}{2}q[A_1 - A_0]\right)(1 - ikA_1)} + \frac{B_0/(B_1 + B_0)}{\left(1 - i\frac{1}{2}q[A_0 - A_1]\right)(1 - ikA_0)} \right|^2 .$$
(3)

Here B_0 and B_1 are the *bare* $pp \to ppK\bar{K}$ amplitudes for producing s-wave $K\bar{K}$ pairs in isospin-0 and 1 states, respectively. These amplitudes, which already include the *fsi* in the K^-p and pp channels [2], are then distorted through a *fsi* corresponding to elastic K^+K^- scattering. This leads to enhancement factors of the form $1/(1 - ikA_I)$, where k is the momentum in the K^+K^- system and A_I is the s-wave scattering length in each of the two isospin channels. The charge-exchange *fsi* depends upon the $K^0\bar{K}^0 \to K^+K^-$ scattering length, which is proportional to the difference between A_0 and A_1, and on the momentum q in the $K^0\bar{K}^0$ system.

A cusp structure might arise because q changes from being purely real above the $K^0\bar{K}^0$ threshold to purely imaginary below this point. The strength of the effect depends upon $A_0 - A_1$, but its shape also depends upon the interference with the direct K^+K^- production amplitude.

There is great uncertainty in the values of the scattering lengths and the choices made in Ref. [4], $A_1 = (0.1\pm0.1)+i(0.7\pm0.1)$ fm and $A_0 = (-0.45\pm$

$0.2)+i(1.63\pm0.2)$ fm, imply a significant charge-exchange contribution. The subsequent fitting of the data on the basis of Eq. (3) is best achieved with $|B_1/B_0|^2 = 0.38^{+0.24}_{-0.14}$, *i.e.*, the kaon pairs are produced dominantly in the isospin-zero combination.

The resulting fit shown in Fig. 2 manifests a cusp at the $K^0\bar{K}^0$ threshold, though the data themselves are not sufficiently precise to see this unambiguously. The other fits shown there are non-allowed solutions, where one neglects either the elastic or charge-exchange *fsi*.

The energy dependence of the total cross section shown in Fig. 3 is definitely improved when the $K\bar{K}$ *fsi* is included but these, and especially the differential data, have to be improved in order to be compelling.
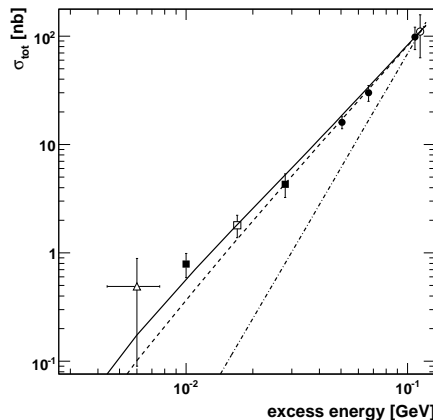

Fig. 3. Experimental total cross sections for $pp \to ppK^+K^-$ as a function of the excess energy. The dot-dashed curve is that of four-body phase space normalised on the 108 MeV point. The dashed curve includes final state interactions between the K^- and the protons and between the two protons themselves [2]. The further consideration of the *fsi* between the kaons leads to the solid curve [4].

In addition to the $pp \to pK^+\{pK^-\}$ measurement, the ANKE collaboration also extracted data on the $pp \to pK^+\{\Sigma^0\pi^0\}$ reaction at 3.65 GeV/c [5]. As a second example of a coupled-channel effect, I would like to argue that both data sets might be understood in terms of the production and decay of the $\Lambda(1405)$, even though this resonance has a nominal mass below the pK^- threshold. To investigate this we have to study the coupled $K^-p \rightleftharpoons \pi^0\Sigma^0$ systems in some detail. This is easiest to achieve within the realm of a separable potential description because the resulting equations can be solved algebraically. Separate and conquer [6] !

A separable description of the $I = 0$ coupled–channel system has been

given in Ref. [7]. Here the potential is taken in the form

$$V_{ij}(p, p') = (2\pi)^3 \frac{A_{ij}}{(p^2 + \beta^2)(p'^2 + \beta^2)} , \qquad (4)$$

which is a symmetric matrix in the two channels (1) $\Sigma\pi$ and (2) $\bar{K}N$.

Define a diagonal matrix of form factors

$$\Pi_{ij} = \frac{1}{(p_i^2 + \beta^2)} \delta_{ij} , \qquad (5)$$

where the momentum p_i in channel i is fixed in terms of the overall c.m. energy W. For the Yamaguchi form factors of Eq. (4), define a second diagonal matrix of dispersion integrals:

$$\Delta_{ij} = \frac{m_i}{4\pi\beta(\beta - ip_i)^2} \delta_{ij} , \qquad (6)$$

where m_i is the reduced mass in channel i.

The Schrödinger equation can then be resolved to give the purely S–wave T–matrix

$$T(W) = \Pi(I + A\Delta)^{-1}A\,\Pi . \qquad (7)$$

The resulting differential cross sections becomes

$$\left(\frac{d\sigma}{d\Omega}\right)_{j \to i} = \frac{m_i m_j}{4\pi^2} \frac{p_i}{p_j} |T_{ij}|^2 . \qquad (8)$$

The available experimental data are fit with the input $I = 0$ potentials [7]

$$A_{11} = -0.176\,\text{fm}^2 , \quad A_{12} = 1.414\,\text{fm}^2 , \quad A_{22} = -1.370\,\text{fm}^2 \qquad (9)$$

with $\beta = 3.5\,\text{fm}^{-2}$. These values lead to a $\Lambda(1405)$ pole at $W = (1406.5 - 25i)$ MeV. The relation between momenta and overall energies was evaluated using non-relativistic kinematics, though this might be questioned for $\pi\Sigma$.

The above formalism is suitable for the description of free coupled $\pi\Sigma/\bar{K}N$ scattering. Suppose now that we introduce a third channel, in this case the initial pp system, that is coupled weakly to these two. In lowest order perturbation theory, the transition matrix element from channel–3 to the other two is then given by

$$\mathcal{T}_i(W) = \left[\Pi(I + A\Delta)^{-1}\right]_{ij} C_j . \qquad (10)$$

The C_j represents a column vector of the initial preparation of the system in the raw $\pi\Sigma/\bar{K}N$ states before the final state interaction is introduced.

In keeping with the assumption of a short-range transition, we neglect any energy or mass dependence of the preparation vector C.

The values of $|\mathcal{T}|^2$ must be multiplied by the phase spaces for $pp \to pK^+\{pK^-\}$ and $pp \to pK^+\{\Sigma^0\pi^0\}$, with a consistent relative normalisation. The shapes of the distributions are determined by the (complex) ratio C_2/C_1. To simplify the notation, we take $C_1 = 1$ and, purely for presentational purposes, normalise each data set to the integrated measured cross sections. We can then ask whether the relative normalisation is as predicted.
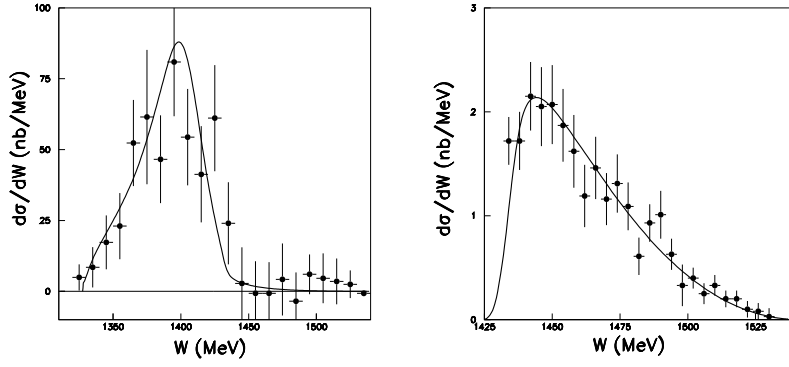

Fig. 4. Cross sections for (left) $pp \to pK^+\{\Sigma^0\pi^0\}$ [5] and (right) $pp \to pK^+\{pK^-\}$ [2] at a beam momentum of 3.65 GeV/c in terms of the $\Sigma^0\pi^0$ and pK^- invariant masses, respectively. In the latter case the contribution from ϕ production was excluded. Theoretical predictions in the separable potential model were obtained with $C = -0.7i$.

Figure 4 is obtained if the purely imaginary value $C_2 = -0.7i$ is used. This value predicts a total cross section ratio

$$R_{K\pi} = \sigma(pp \to pK^+\{pK^-\})/\sigma(pp \to pK^+\{\Sigma\pi\}^0) = 9.4 \times 10^{-3}, \quad (11)$$

to be compared to the experimental value of $(22 \pm 8) \times 10^{-3}$, where the contribution from ϕ production is not included. This is perfectly acceptable agreement, given the model's simplicity. Apart from other defects, the pp *fsi* has been neglected, as has any quantum mechanical interference arising from the presence of two final protons.

There is one rather tricky point that must be mentioned. In order to get good agreement for the shape of the K^-p spectrum, Maeda *et al.* [2] needed to put in the *fsi* of the K^- with *both* protons. In the present approach it is assumed that the whole K^-p distribution shown does in fact come from

the $\Lambda(1405)$ channel, even though there are two protons in the final state. This is in fact completely consistent with the factorisation assumption that the K^- can have simultaneous *fsi* with both protons.

The obvious question now is: "How stable are the results to changes in the value of C_2?". The short answer is: "not very!". This is illustrated in Fig. 5, where a purely real value $C_2 = +0.5$ is chosen. The K^-p spectrum doesn't change too much (although it looks as though my hand must have been shaking when I drew it) but the $\Sigma\pi$ can vary enormously. For this value of C_2 one can even generate a double-peaked structure. The predicted cross section ratio of 18×10^{-3} is close to experiment, but that is pretty meaningless in view of the complete failure to describe the shape.
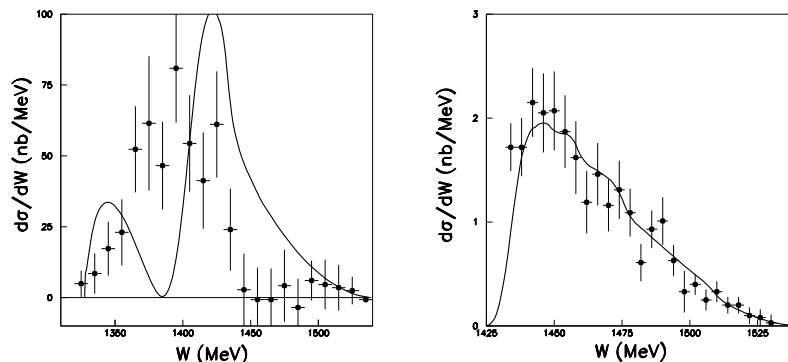

Fig. 5. Cross sections for (left) $pp \rightarrow pK^+\{\Sigma^0\pi^0\}$ [5] and (right) $pp \rightarrow pK^+\{pK^-\}$ [2], as in Fig. 4. Theoretical predictions were obtained with $C = +0.5$.

The results presented here are still preliminary, and no attempt has been made to include any contribution from the production of isovector K^-p pairs to the cross section. Nevertheless some general conclusions might be drawn. The first, fairly obvious one, is that the ratio of the $pp \rightarrow pK^+\Sigma^0\pi^0$ and $pp \rightarrow pK^+pK^-$ comes out about right in this very hand-waving approach. As a consequence it seems likely that the same underlying reaction mechanism drives both processes and, hence, that one should try to estimate the two cross sections together in a realistic dynamical model.

Following from the above argument, the fact that $K\bar{K}$ scalar resonances cannot contribute in a major way to $pp \rightarrow pK^+\Sigma^0\pi^0$, means that they are unlikely to do so for $pp \rightarrow pK^+pK^-$ either, though they could distort the K^+K^- spectrum at low invariant masses through a *fsi* [4].

Why is the K^-p spectrum fairly stable to changes in the preparation vector C_i while the $\Sigma^0\pi^0$ distribution can change dramatically? The origin

of this probably lies in the form of the separable potential used to describe the channel coupling. Gal [8] points out that the $\Sigma\pi$ diagonal interaction used in Ref. [7] is much weaker than that of the chiral perturbation theory approaches [9]. In other words, the $\Sigma\pi$ is really being driven here more by the K^-p. To check this we would really need a separable potential fitted to the chiral perturbation amplitudes.

Finally we turn to the related question of whether the $\Lambda(1405)$ is actually a single resonance or whether there are two closely spaced states that might be coupled differently to different channels. The ANKE $pp \to pK^+\Sigma^0\pi^0$ data show no sign of any two-peak structure [5] but Geng and Oset [10] have shown that this is not necessary or even likely in a two-pole scenario. It depends on the background and on how the state is prepared. In a sense, this is also what is found here in a much more intuitive approach. The shape of the spectra will depend upon the preparation vector as well on as the coupling potentials. In brief, two poles do not necessarily imply two peaks and two peaks do not necessarily imply two poles!

Coupled-channel effects in strange particle production at intermediate energies seem to be a rich field for theorists to till in the next few years provided that we are given more data, preferably with higher statistics!

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